Lecture (02) Electric Charges and Electric Field (2)

By:

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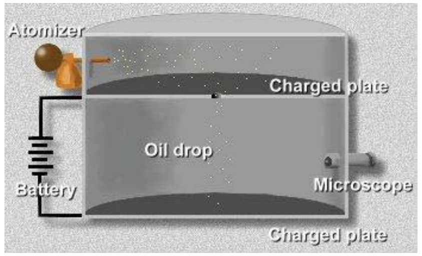
Charge

- What is charge?
  - Electron: Considered a point object with radius less than 10^{-18} meters with electric charge e= -1.6×10^{-19} Coulombs (SI units) and mass $m_e= 9.11 \times 10^{-31}$ kg
- How do we visualize it?
  - We only know charge exists because in experiments electric forces cause objects to move.
- Charge is analogous to mass in mechanics, We know how it behaves, but we don't know what it really is.
- The same is true for charge.

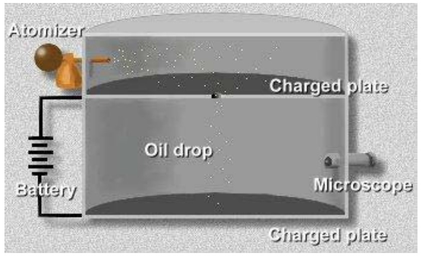
quantization of charge

- Discovered in 1911 by Robert A. Millikan in the oil drop experiment
- The unit of charge is so tiny that we will never notice it comes in indivisible lumps.

- An experiment performed by Robert Millikan in 1909 determined the size of the charge on an electron.
- He also determined that there was a smallest 'unit' charge, or that charge is 'quantized'.
- He received the Nobel Prize for his work.
- What Millikan did was to put a charge on a tiny drop of oil, and measure how strong an applied electric field had to be in order to stop the oil drop from falling.



- Since he was able to work out the mass of the oil drop, and he could calculate the force of gravity on one drop, he could then determine the electric charge that the drop must have.
- By varying the charge on different drops, he noticed that the charge was always a multiple of -1.6×10^{-19} C, the charge on a single electron.
- This meant that it was electrons carrying this unit charge.



- When a drop is suspended, its weight $m \cdot g$ is exactly equal to the electric force applied $q \cdot E$

- The values of E, the applied electric field, m the mass of a drop, and g, the acceleration due to gravity, are all known values. So you can solve for q, the charge on the drop:

$$q \cdot E = m \cdot g$$
$$q = \frac{m \cdot g}{E}$$

- The charge q on a drop was always a multiple of -1.6×10^{-19} C, the charge on a single electron.

Example 00

- Suppose in a typical experiment we charge an object up with a nanoCoulomb of charge ($Q = 10^{-9}$ C).
- How many elementary units, N, of charge is this?

- Elementary unit of charge $e = +1.6 \times 10^{-19}$ *C*

$$N = \frac{Q}{e} = \frac{10^{-9}\,C}{+1.6 \times 10^{-19}\,C} = 0.625 \times 10^{10} = 6.25 \times 10^{9}$$

- Six billion units of charge

Coulombs Law

- In 1785 Charles Augustin Coulomb reported two charged bodies repelled each other with a force that is inversely proportional to the distance, and proportional to quantity of charges.

$$F = k\frac{q_1 q_2}{r^2} \quad \text{where } k = 8.99\times 10^9 \text{Nm}^2/\text{C}^2$$

$$\text{Also } F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \text{where } \varepsilon_0 = 8.85\times 10^{-12}\ \text{C}^2/N.m^2$$

Electrical Permittivity of vacuum

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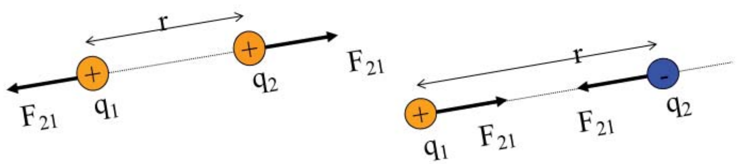
$$[F] = [k_e]\frac{[q_1][q_2]}{[r]^2}$$

$$[Newton] = [k_e]\frac{[Coulomb][Coulomb]}{[meter]^2}$$

$$[k_e] = N\cdot m^2/C^2$$

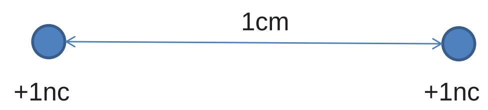
- The electrostatic force is often called Coulomb force.
- It is a force:
  - a magnitude
  - a direction.



- Second example of action at a distance.

Example 01

- What is the magnitude of the force between two positive charges, each 1 nano Coulomb, and 1cm apart in a typical demo?



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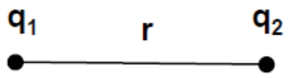
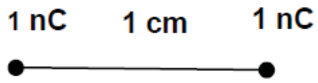


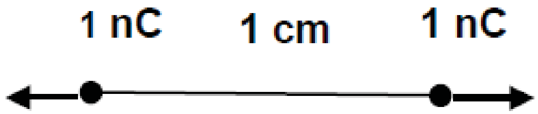


$$F=\frac{kq_1q_2}{r^2}$$

$$F=\frac{\left(8.99\times10^{9}\,\frac{Nm^2}{c^2}\right)\left(10^{-9}c\right)^2}{\left(10^{-2}m\right)^2}=9\times10^{-5}N$$

(Equivalent to the weight of a long strand of hair)

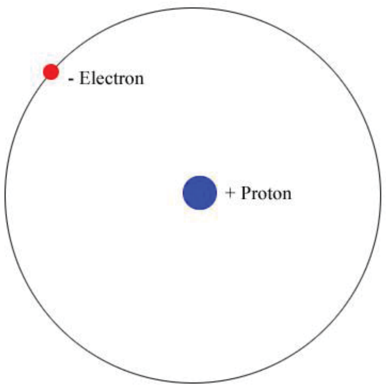
- What is the direction of the force?

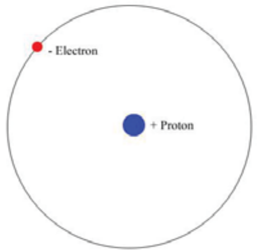


Repulsion

Example 02

- The electron and proton of a hydrogen atom are separated (on the average) by a distance of about 5.3x10^{-11} m. Find the magnitude of the electric force that each particle exerts on the other.





- q_1 =-1.60x10^{-19} C
- q_2 =1.60x10^{-19} C
- r = 5.3x10^{-11} m

$$F_e=k_e\frac{|e|^2}{r^2}=8.99\times10^{9}\,\frac{Nm^2}{C^2}\,\frac{\left(1.6\times10^{-19}C\right)^2}{\left(5.3\times10^{-11}m\right)^2}=8.2\times10^{-8}N$$

- Attractive force with a magnitude of 8.2x10^{-8} N.

Example 03

- Which charge exerts the greater force? Two positive point charges, Q_1 = *50 uC* and Q_2 = 2 *uC,* are separated by a distance *l.*
- Which is larger in magnitude, the force that Q_1 exerts on Q_2 *,or* the force that Q_2 exerts on Q_1*?*

- From Coulomb's law, the force on *Q1* exerted by *Q1* is

$$F_{12} = k\frac{Q_1 Q_2}{\ell^2}.$$

- The force on *Q2* exerted by *Q1* is

$$F_{21} = k\frac{Q_2 Q_1}{\ell^2}$$

- which is the same magnitude. The equation is symmetric with respect to the two charges, so $F_{21} = F_{12}$.
- NOTE Newton's third law also tells us that these two forces must have equal magnitude.

Example 04

- In previous example, Calculate force if l = 30 cm

30 cm

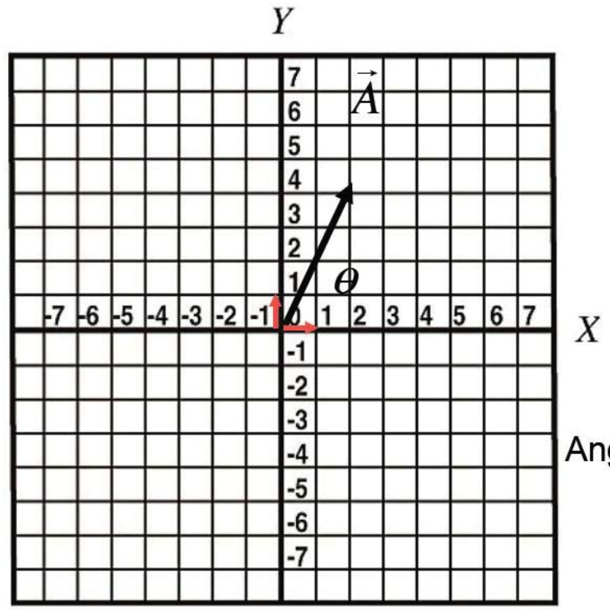
- F = $k\ \frac{Q_1 \times Q_2}{r^2} =$

$8.99 \times 10^9 \times 50 \times 10^{-6} \times 1 \times 10^{-6} / (0.3)^2 = 14.98$ N

Overview on vectors

- Representation of a vector : has magnitude and direction
  - i and j unit vectors
  - x and y components
  - angle gives direction and length of vector gives the magnitude

Vectors



$\vec{A} = \langle 2\hat{i} + 4\hat{j} \rangle$

Red arrows are the i and j unit vectors.

Magnitude =

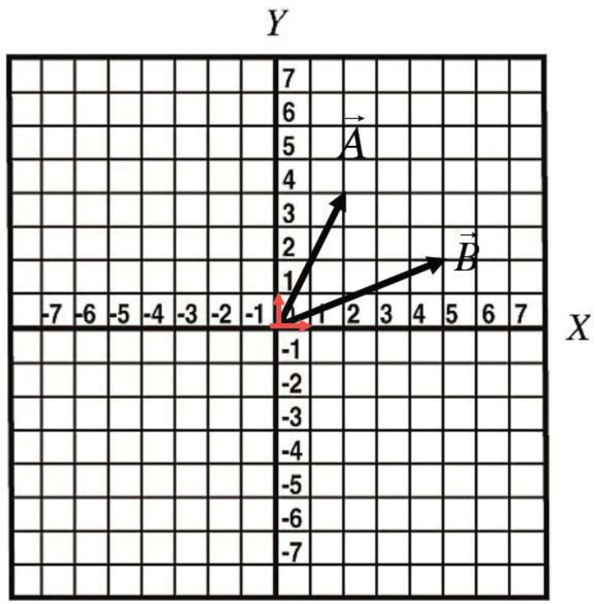
$|A| = \sqrt{2^2 + 4^2} = \sqrt{20} = 4.47$

Angle between A and x axis =

$\tan\theta = y / x = 4 / 2 = 2$

$\theta = 63.4 \deg$

Adding Two Vectors

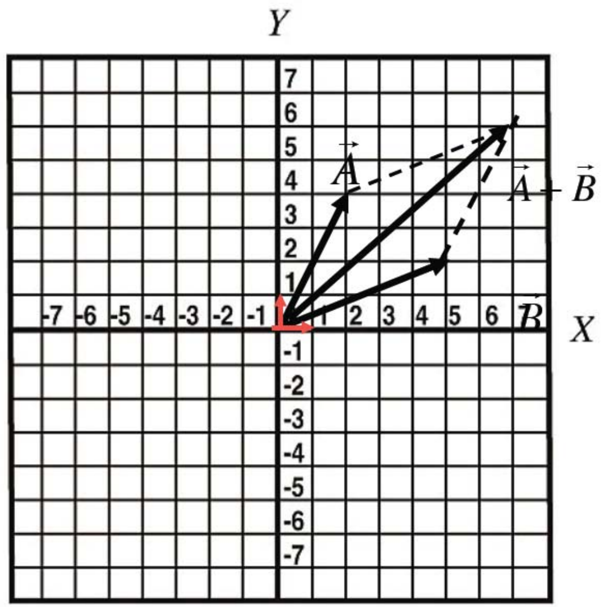


$\vec{A} = \langle 2\hat{i} + 4\hat{j} \rangle$

$\vec{B} = \langle 5\hat{i} + 2j \rangle$

Create a Parallelogram with The two vectors You wish you add.

Adding Two Vectors



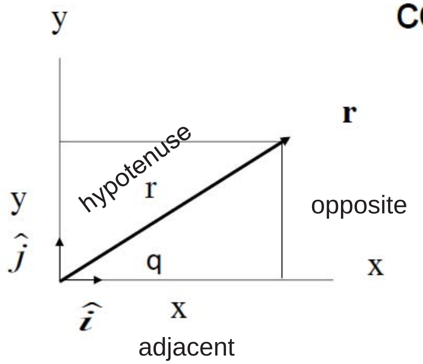
$\vec{A} = \langle 2\hat{i} + 4\hat{j} \rangle$

$\vec{B} = \langle 5\hat{i} + 2j \rangle$

$\vec{A} + \vec{B} = \langle 7\hat{i} + 6\hat{j} \rangle$

Note you add x and y components

Vector components in terms of sine and cosine



$$\cos\theta = x/r \qquad x = r\cos\theta$$

$$\sin\theta = y/r \qquad y = r\sin\theta$$

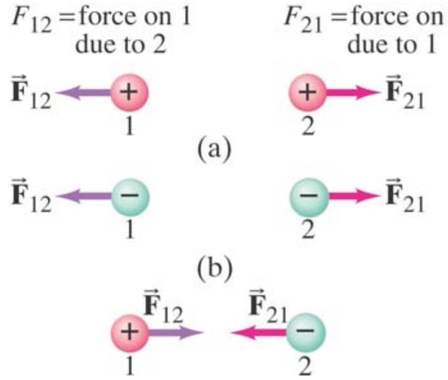
$$r = x\hat{i} + y\hat{j}$$

$$r = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}$$

$$\tan\theta = y / x$$

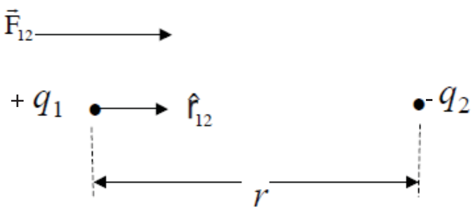
Coulombs Law in vector notation inline charges

- The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.



Newton's Third Law applies!

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

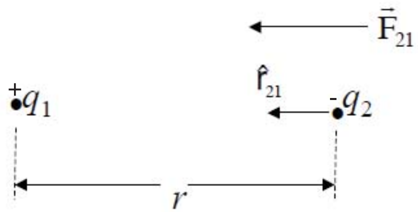


where:

$\vec{F}_{12}$ is the force exerted by particle 1 on particle 2,

$\hat{r}_{12}$ is a unit vector in the direction "from 1 to 2", and

k, q_1, and q_2 are defined as before (the Coulomb constant, the charge on particle 1, and the charge on particle 2 respectively).



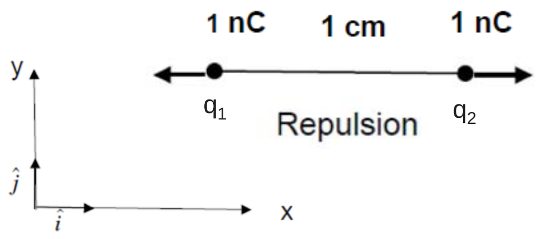
where:

$\vec{F}_{21}$ is the force exerted by particle 2 on particle 1,

$\hat{r}_{21}$ is a unit vector in the direction "from 2 to 1", and

k, q_1, and q_2 are defined as before (the Coulomb constant, the charge on particle 1, and the charge on particle 2 respectively).

- Return to previous example : What is the direction of the force? In vectors notation,..



$F_1 = -9\times10^{-5}\,N\ \hat{i}$

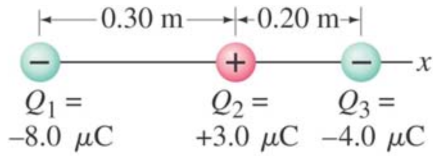
$F_2 = +9\times10^{-5}\,N\ \hat{i}$

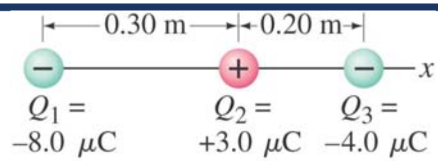
$$F = \frac{\left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\left(10^{-9}\,c\right)^2}{\left(10^{-2}\,m\right)^2} = 9\times10^{-5}\,N$$

Example 5

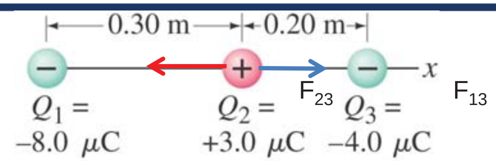
Three charges in a line.

- Three charged particles are arranged in a line.
- Calculate the net electrostatic force on particle 2 (the +3.0 µC in the middle) due to the other two charges.

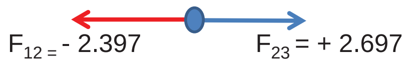




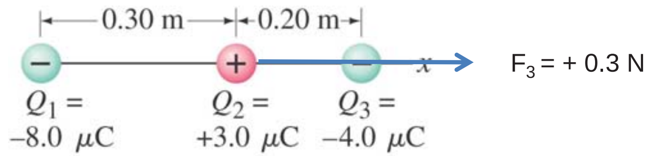
- Coulomb's law gives the magnitude of the forces on particle 2 from particle 3 and from particle 2.
- The directions of the forces can be found from the geometrical arrangement of the charges



- F_{21} = - 8.99 x 10^9 x $(8x10^{-6}x3x10^{-6})/(.3)^2$ = -2.397 N
- F_{23} = 8.99 x 10^9 x $(3x10^{-6}x4x10^{-6})/(.2)^2$ = +2.697 N

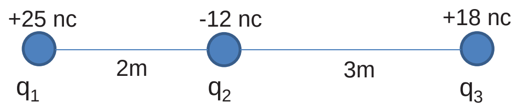


- F_2 = + 0.3N



Example 06

- Three charges lie on the x axis: q1=+25 nC at the origin, q2= -12 nC at x = 2m, q3=+18 nC at x=3 m. What is the net force on q1 and the direction of the force?



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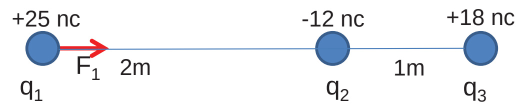
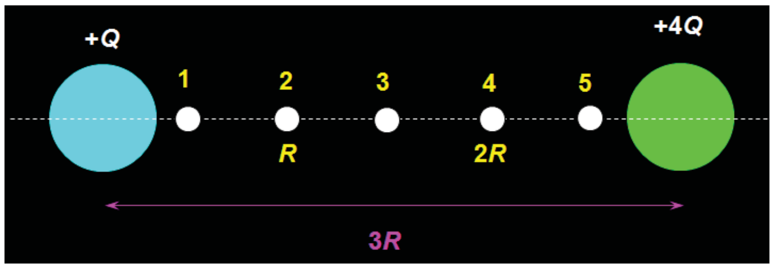
- $\vec{F}_{12}$ = +8.99 x 10^9 x (25 x 10^{-9} x 12 x 10^{-9})/$(2)^2$ = +6.7425x 10^{-7} N
- $\vec{F}_{13}$ = -8.99 x 10^9 x (25x 10^{-9} x 18 x 10^{-9})/$(3)^2$ = -4.495 x 10^{-7} N
- $\vec{F}_{1}$ = 6.7425 x 10^{-7} – 4.495 x 10^{-7} = 2.2475 x 10^{-7} N



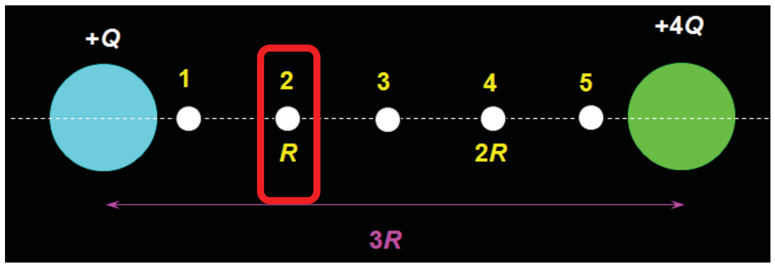
MCQ – Example 07

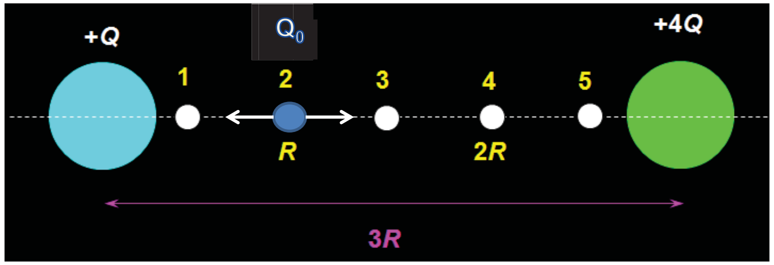
- Two balls with charges +Q and +4Q are separated by 3R. Where should you place another charged ball Q_0 on the line between the two charges such that the net force on Q_0 will be zero?



MCQ solution

- Coulombs low: $F = k(Q_1)(Q_2)/R^2$
- Since +4*Q* is 4 times bigger than +*Q*, Q_0 needs to be farther from +4*Q*. In fact, Q_0 must be twice as far from +4*Q*, since the distance is squared in Coulomb's law.
- So Q_0 should be placed on position 2

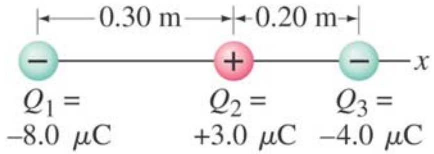


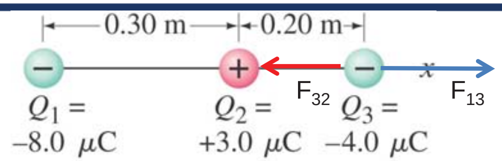


- $F_{04} = + K (Q_0 \times 4 \times Q)/(4 \times R^2)$
- $F_{01} = - K (Q_0 \times Q)/(R^2)$
- $F_0 = F_{04} – F_{01} = K(Q_0 \times Q) /R – K(Q_0 \times Q)/R = 0$

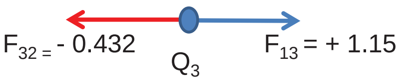
Example 8

- Calculate the net electrostatic force on particle 3

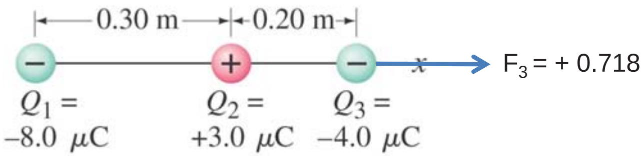




- $F_{31} = + 8.99 \times 10^9 \times (8\times10^{-6}\times4\times10^{-6})/(.5)^2 = 1.15$ N
- $F_{32} = - 8.99 \times 10^9 \times (3\times10^{-6}\times4\times10^{-6})/(.2)^2 = - 0.432$ N



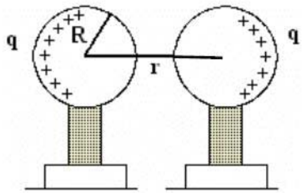
- $F_3 = + 0.718$ N



Uniformly charged metal spheres of Radius R

- $F = K \times q_1 \times q_2 / r^2$

- $F == K \times q_1 \times q_2 / (r + 2R)^2$

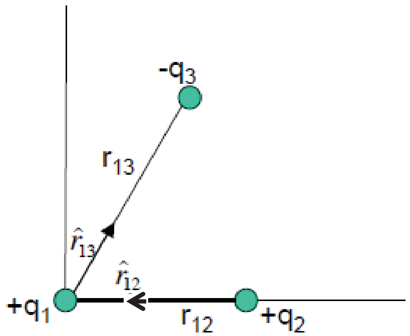


Coulombs Law in vector notation angular charges

- What is the force on charge q1 due to many other charges lying in a plane

$$\vec{F}_{12} = \frac{kq_1q_2}{(r_{12})^2}\hat{r}_{12}$$

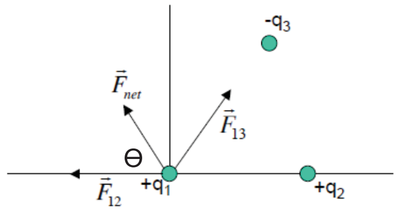
$$\vec{F}_{13} = \frac{kq_1q_3}{(r_{13})^2}\hat{r}_{13}$$



- In practice you find the x and y components of all the vectors and add them up

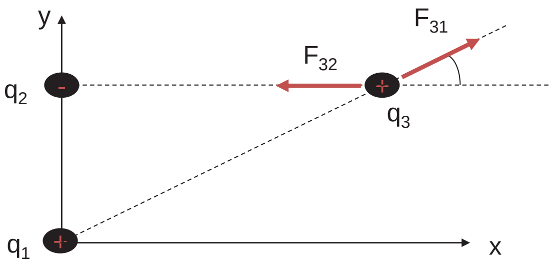
$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13} \qquad \vec{F}_{net} = \sum_{i=2}^{N} \vec{F}_{1i}$$

$$\tan\theta = \frac{F_{net\,y}}{F_{net\,x}}$$



Superposition Principle

- From observations: one finds that whenever multiple charges are present, the net force on a given charge is the *vector* sum of all forces exerted by other charges.
- Electric force obeys a ***superposition principle***.



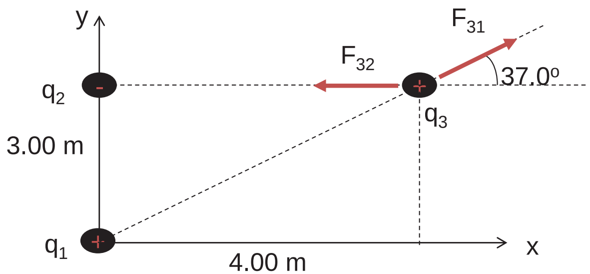
Example 09

Consider three point charges at the corners of a triangle, as shown below. Find the resultant force on q_3 if

$q_1 = 6.00 \times 10^{-9}$ C

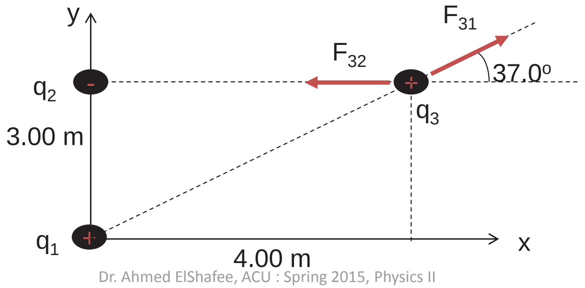
$q_2 = -2.00 \times 10^{-9}$ C

$q_3 = 5.00 \times 10^{-9}$ C



Observations:

- The superposition principle tells us that the net force on q_3 is the vector sum of the forces F_{32} and F_{13}.
- The magnitude of the forces F_{32} and F_{13} can calculated using Coulomb's law.



- Consider three point charges at the corners of a triangle, as shown below. Find the resultant force on q_3.

$$F_{32}=k_e\frac{|q_3||q_2|}{r^2}=8.99\times10^9\,\tfrac{Nm^2}{C^2}\frac{(5.00\times10^{-9}C)(2.00\times10^{-9}C)}{(4.00m)^2}=5.62\times10^{-9}N$$

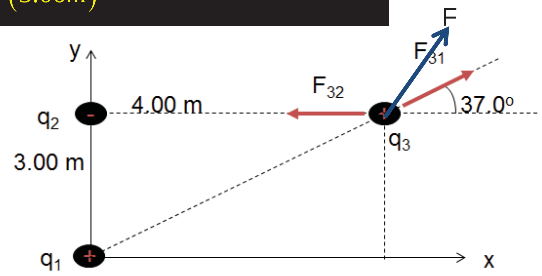
$$F_{31}=k_e\frac{|q_3||q_1|}{r^2}=8.99\times10^9\,\tfrac{Nm^2}{C^2}\frac{(5.00\times10^{-9}C)(6.00\times10^{-9}C)}{(5.00m)^2}=1.08\times10^{-8}N$$

$$F_x=F_{32}+F_{31}\cos 37.0^o=3.01\times10^{-9}N$$

$$F_y=F_{31}\sin 37.0^o=6.50\times10^{-9}N$$

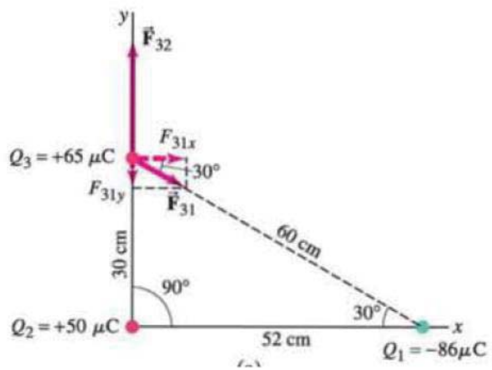
$$|F|=\sqrt{F_x^2+F_y^2}=7.16\times10^{-9}N$$

$$\theta=65.2^o$$



Example 10

- Calculate the net
- electrostatic force on charge *Q3*, due to the charges *Q1* and *Q2*.



F_{23}

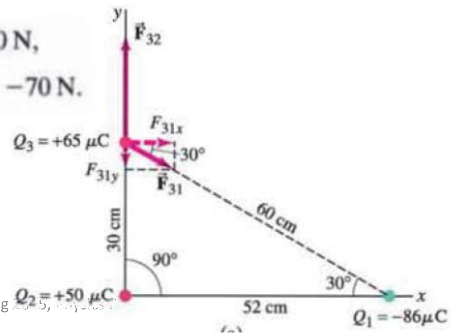
$$F_{31}=k\frac{Q_3Q_1}{r_{31}^2}=\frac{(9.0\times10^9\,N\cdot m^2/C^2)(6.5\times10^{-5}\,C)(8.6\times10^{-5}\,C)}{(0.60\,m)^2}=140\,N,$$

$$F_{32}=k\frac{Q_3Q_2}{r_{32}^2}=\frac{(9.0\times10^9\,N\cdot m^2/C^2)(6.5\times10^{-5}\,C)(5.0\times10^{-5}\,C)}{(0.30\,m)^2}=330\,N.$$

- We resolve F31 into its components along the *x* and y axes,

$$F_{31x}=F_{31}\cos 30^\circ=(140\,N)\cos 30^\circ=120\,N,$$

$$F_{31y}=-F_{31}\sin 30^\circ=-(140\,N)\sin 30^\circ=-70\,N.$$



- The force $\vec{F}_{32}$ has only a y component. So the net force $\vec{F}$ on Q_3 has components

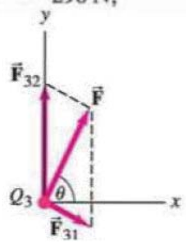
$$F_x = F_{31x} = 120\,\text{N},$$
$$F_y = F_{32} + F_{31y} = 330\,\text{N} - 70\,\text{N} = 260\,\text{N}.$$

The magnitude of the net force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(120\,\text{N})^2 + (260\,\text{N})^2} = 290\,\text{N};$$

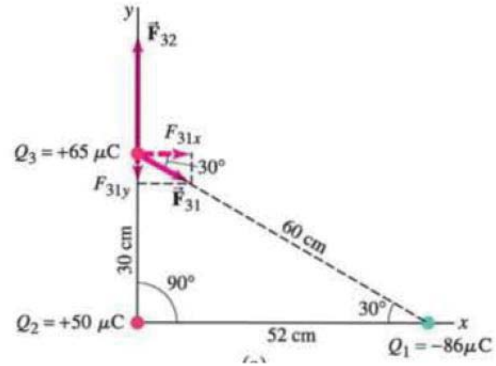
$$\frac{F_y}{F_x} = \frac{260\,\text{N}}{120\,\text{N}} = 2.2,$$

$$\theta = \tan^{-1}(2.2) = 65°.$$

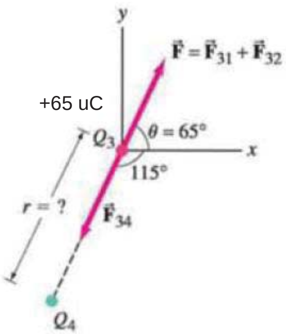


Example 11

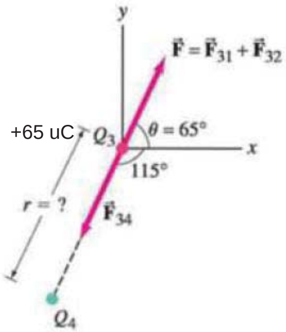
- Make the force on Q3 zero. where could you place a fourth charge, *Q4* = -50 uC, so that the net force on Q3 would be zero?



- By the principle of superposition, we need a force in exactly the opposite direction to the resultant F due to Q2 and *Q1* that we calculated in last example.
- Our force must have magnitude <u>290N</u>, and must point down and to the left of Q3.
- So *Q4* must be along this line.

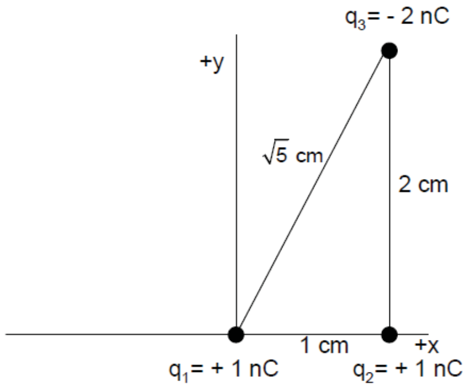


- F_{34} = 290 N = 8.99 x 10^9 x (65 x 10^{-6}) x (50 x 10^{-6}) / r^2
- r_{34} = .317 m

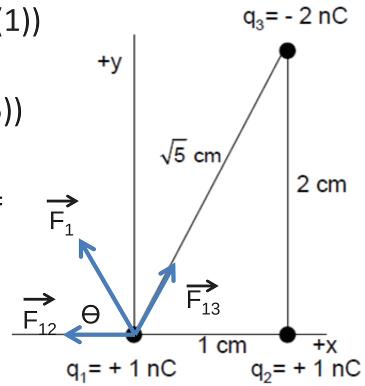


Example 12

- What is the net force on q1 and in what direction?

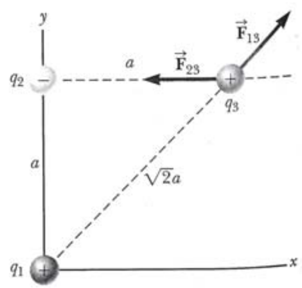


- $\vec{F}_{12}$ = - (8.99 x 10^9 x (1 x 10^{-9} x 1 x 10^{-9})/(1)) = - 8.99 x 10^{-9}
- $\vec{F}_{13}$ = (8.99 x 10^9 x (1 x 10^{-9} x 2 x 10^{-9})/(5)) = (3.6 x 10^{-9}) N
- $\vec{F}_{1x}$ = - 8.99 x 10^{-9} + 3.6 x 10^{-9} x (1/$5^{0.5}$) = -7.36 x 10^{-9}
- $\vec{F}_{1y}$ = (3.6 x 10^{-9}) x (2/$5^{0.5}$) = 3.22 x 10^{-9}
- |F1| = ((7.36 x $10^{-9})^2$ + (3.22 x $10^{-9})^2$ $)^{0.5}$
  = 8.034 x 10^{-9} N
- ϴ = $\tan^{-1}$ (+ (3.22 x 10^{-9}) / (-7.36 x 10^{-9})) =
  = - 26.06 °



Example 13

- What is the net force on q3 and in what direction?

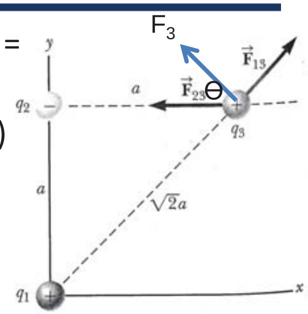


$q_1 = q_3 = 5.00\ \mu C$

$q_2 = -2.00\ \mu C$

$a = 0.100\ m$

- $\vec{F}_{13}$ = 8.99 x 10^9 x (5 x 10^{-6} x 5 x 10^{-6})/(0.02) = 11.24 N
- $\vec{F}_{23}$ = - 8.99 x 10^9 x (5 x 10^{-6} x 2 x 10^{-6})/(0.01) = 8.99 N
- $\vec{F}_{3x}$ = +11.24 x (.1/($2^{0.5}$x.1)) – 8.99 = 7.95 – 8.99 = -1.42 N
- $\vec{F}_{3y}$ = = +11.24 x (.1/($2^{0.5}$x.1)) = 7.95 N
- ϴ = $\tan^{-1}$ (7.99 / -1.42) = - 80 °



$q_1 = q_3 = 5.00\ \mu C$

$q_2 = -2.00\ \mu C$

$a = 0.100\ m$


Thanks,..
See you next week (ISA),…
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Dr. Ahmed ElShafee, ACU : Spring 2015, Physics II